TRINITY COLLEGE

Mathematics Applications Unit 3/4
Test 5 2020

Calculator Assumed
Finance

STUDENT'S NAME Marking Key

DATE: Friday 14th August **TIME:** 55 minutes **MARKS**: 52

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (7 marks)

(a) What value must be invested into an account paying 5% simple interest if it accrues \$2390 interest over 8 years [2]

$2390 = \dfrac{P \times 5 \times 8}{100}$ ✓

$P = \$5975$ ✓

(b) Calculate, to 2 decimal places, the annual compound interest rate required to see an initial investment of \$20 000 grow to \$48 000 in 10 years. [2]

$48\,000 = 20\,000\left(1 + \frac{r}{100}\right)^{10}$ ✓

$r = 9.15\%$ ✓

(c) Darren invests \$650 000 in an account that returns 4.2% p.a. compounded quarterly and withdraws an annual perpetuity. How much will Darren receive each year from this investment? [3]

N = 1
I = 4.2
PV = −650 000
PMT =
FV = 650 000
P/Y = 1
C/Y = 4

PMT = 27732.99

\$27732.99

✓ P/Y + C/Y correct
✓ All other variab
✓ Solution

2. (6 marks)

To save money for a car, Josh started an investment account which accrues 9.6% interest p.a. He placed an initial deposit of $6000, and then deposited an extra $200 at the end of each month for a year.

The table below shows the amount in the account at the beginning of each month (A_n), the interest added to the account each month (I_n), the deposit made at the end of each month (D_n), and the amount in the account at the end of each month (A_{n+1}) for the first 6 months.

Month (n)	**Amount at beginning of month (A_n)**	**Interest for month (I_n)**	**Deposit for month (D_n)**	**Amount at end of month (A_{n+1})**
1	$6000.00	$48.00	$200.00	$6248.00
2	$6248.00	$49.98	$200.00	$6497.98
3	$6497.98	$51.98	$200.00	$6749.97
4	$6749.97	$54.00	$200.00	$7003.97
5	$7003.97	$56.03	$200.00	$7260.00
6	$7260.00	$58.08	$200.00	$7518.08

(a) What is the monthly interest rate? [1]

$\frac{9.6}{12} = 0.8\%$ ✓

(b) Write a recursive rule to determine the amount in the account at the end of each month. [2]

$A_{n+1} = 1.008 \cdot A_n + 200$ $A_0 = 6000$

✓ rate = 1.008

✓ A_0 & +200

(c) What is the amount in the account at the end of the 12 months? [1]

$A_{12} = 9110.50$

$9110.50 ✓

(d) What is the total amount of interest earned over the course of the year? [2]

$9110.5 - 6000 - 12(200)$

✓ subtract 12 × 200

✓ subtract 6000 & calc ans

= $710.5

3. (7 marks)

Bailey was selected to play on a TV game show. He wins a round called "Higher or Lower", the prize for which was a 30 m yacht. The recursive formula $B_{n+1} = 0.84B_n$, $B_0 = 580000$ can be used to calculate the value of the yacht after n years.

(a) What is the significance of $B_0 = 580000$? [2]

✓ state B_0 is initial value

✓ indicates \$580000

(b) State the annual rate of depreciation of the yacht. [1]

16% ✓

(c) To the nearest thousand dollars, what is the value of his yacht after 4 years? [2]

$B_4 = 288765.39$ ✓ indicates term 4

$= \$289\,000$ ✓ calculates ans to nearest \$1000

(d) When the value of the yacht has fallen below \$200 000, Bailey sells the yacht. For how many years did he have the yacht? [2]

$B_6 = 203752.86$

✓ states B_6 & B_7

$B_7 = 171152.40$

✓ states 7 yrs.

∴ 7 years

4. (8 marks)

A sum of $100 000 was borrowed at an interest rate of 12% p.a. It was agreed that the loan had to be repaid by equal investments over 20 years. The repayment options were as follows and interest is compounded at the same frequency as the repayments:

Option 1	Annual repayments of $13387.88
Option 2	Quarterly repayments of $3311.17
Option 3	Monthly repayments of $1101.09

(a) Calculate, for each option, the total amount that must be paid back to the loan and state the cheapest option. [3]

(1) 13387.88 x 20 = $267757.60

(2) 3311.17 x 4 x 20 = $264893.60

(3) 1101.09 x 12 x 20 = $264261.6

∴ Option 3 is cheapest

✓ 2 correct totals ✓ indicates which is cheapest

✓ all correct totals

(b) For the option chosen in part (a), calculate the balance owing after

(i) 2 years. [2]

N = 24
I = 12
PV = 100 000
PMT = -1101.09
FV = ?
P/Y = 12
C/Y = 12

FV = -97273.25

$97273.25

✓ correct input

✓ final ans.

(ii) 20 years. [2]

N = 240

"

FV = 3.82

$3.82

✓ correct N value

✓ final ans.

(c) Explain why the amount owing after 20 years is not $0. [1]

The last payment is $3.82 less than all others.

✓ any reasonable answer.

5. (8 marks)

Aiden plans to complete a university course over a period of 3 years. He estimates that he will require $250 per week over the three years to cover living expenses, which he will draw from an investment account that accrues 7.5% p.a. compounded weekly.

(a) How much, to the nearest thousand dollars, must he initially deposit into the account to cover the cost of his living expenses? [2]

N = 156
I = 7.5
PV =
PMT = 250
FV = 0
P/Y = 52
C/Y = 52

PV = -34 901.4

$35 000

✓ correct input

✓ final ans

(b) How much will Aiden withdraw from the account over the 3-year period. [1]

$250 \times 156 = \$39\,000$ ✓

(c) Write a recurrence relation that will allow Aiden to keep track of his weekly balances in the account. [3]

$T_{n+1} = \left(1 + \frac{0.075}{52}\right) T_n - 250$

$T_0 = 35000$

✓ rate
✓ withdrawl
✓ T_0

(c) What is the balance of Aiden's account halfway through his first year of university? [2]

$T_{26} = 29718.89$

✓ indicates 26^{th} term

$29718.89

✓ final ans.

6. (16 marks)

Dylan has a mortgage of \$600 000 on his luxury apartment. The table given below shows the state of his reducible balance mortgage account for month 117 of his loan. Assume that the interest rate remains unchanged throughout the life of the loan. Dylan repays \$6000 per month.

Month	Initial Amount Owing	Interest Added	Repayment	Final Amount Owing
117	\$316 516.02	\$2347.29	\$6000	\$312 863.52

(a) Calculate the annual interest rate charged, rounded to 2 decimal places. [2]

$\frac{2347.29}{316\,516.02} \times 100 \times 12$

$= 8.90\%$

✓ correct numerator & denuminator

✓ multiply by 100 & 12 and calculate final ans.

(b) How much is the principal reduced over the 118th month? [2]

$6\,000 - 312\,863.52 \times \frac{8.90}{100 \times 12}$

$= \$3679.60$

✓ calculates interest from 118th month

✓ subtract from 6000 and calculate final ans.

(c) Determine in which month Dylan will reduce the amount owing to less than half the original amount borrowed and the value of the account at the end of this month. [3]

N =
I = 8.9
PV = 600000
PMT = -6000
FV = -300 000
P/Y = 12
C/Y = 12

N = 120.46

during month 121

N = 121

FV = -297 980.58

\$297 980.58

✓ correct input

✓ states month 121

✓ calculates value at 121

In the month that Dylan reduces the loan to less than half its original value he loses his job in the pandemic and from the next month converts his loan to an "interest only" non-reducible loan. This means his monthly repayments are just enough to cover the interest charged for that month.

(d) What is the monthly repayment that Dylan makes during this time? [2]

$297980.58 \times \frac{8.9}{100 \times 12}$

$= \$2210.02$

✓ uses final value from (c)

✓ multiplies by monthly interest and calculates amount

After 6 months of "interest only" payments, Dylan is again able to make reducible balance repayments, now $7000 per month to his loan.

(e) How long will it take Dylan to pay off the loan entirely? [3]

N =
I : 8.9
PV = 297980.58
PMT: −7000
FV : 0
P/Y: 12
C/Y: 12

N = 51.34

121 + 6 + 52

= 179

✓ calculates repayments @ 7000 = 52

✓ adds 121

✓ adds 6 and calcs final value

(f) What will be the value final repayment of the loan? [2]

N = 52

FV = 4597.23

7000 − 4597.23

$2402.77

✓ calcs excess payment

✓ subtract from 7000 and state solution

(g) Calculate the total interest Dylan will pay on the loan. [2]

$121 \times 6000 + 6 \times 2210.02 + 51 \times 7000 + 2402.77 - 600000$

$= \$498662.89$

✓ calculates two correct repayment periods

✓ calculates all correct repayment periods, subtracts 600 000 and determines final solution